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NON RELATIVISTIC BOUND STATE SOLUTION OF THE SCHRODINGER EQUATION WITH SCREENED COULOMB POTENTIAL

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Abstract

This study investigates the non-relativistic bound-state solutions of the Schrödinger equation for a particle interacting with the screened Coulomb (Yukawa) potential. The screened Coulomb potential modifies the classical Coulomb interaction by introducing an exponential screening parameter that accounts for environmental effects such as plasma shielding and many-body interactions. Due to the mathematical complexity introduced by the screening term, exact analytical solutions of the Schrödinger equation are difficult to obtain. Therefore, the Nikiforov–Uvarov (NU) method was employed to obtain approximate analytical expressions for the energy eigenvalues and corresponding wavefunctions. The radial Schrödinger equation was transformed into a hypergeometric-type differential equation suitable for the NU formalism. Numerical calculations of the energy eigenvalues were performed for different values of the radial quantum number, angular momentum quantum number, and screening parameter. The results show that the screening parameter significantly influences the bound-state energy spectrum, as increasing screening reduces the strength of the effective interaction. The study demonstrates the effectiveness of the Nikiforov–Uvarov method in solving quantum mechanical problems involving screened potentials.

Keywords: Nikiforov–Uvarov method; screened Yukawa potential; Schrödinger equation

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1. Introduction

The non-relativistic Schrödinger equation [1] has been solved for a wide range of central potentials, each chosen to approximate inter-particle interactions under different physical conditions.

$$\frac{d^2u(r)}{dr^2}+\left[\frac{2\mu}{\hbar^2}(E-V(r))-\frac{l(l+1)}{r^2}\right]u(r)=0 \qquad (1)$$

Early quantum theorists such as Schrödinger, Heisenberg [2,3], and Dirac recognized that only a few potentials including the Coulomb and harmonic oscillator, allow exact closed-form solutions[4]. The Coulomb potential in particular has been fundamental in describing hydrogenic atoms, enabling analytic expressions for all energy states.

The potential is expressed as:

$$V(r)=-\frac{V_0e^{-\delta r}}{r} \qquad (2)$$

where V_0 represents the strength of the Coulombic interaction, and δdenotes the screening parameter. The screening parameter quantifies the extent to which surrounding charges weaken the effective interaction. When $\delta=0$, the potential reduces to the unscreened Coulomb form; increasing δleads to rapid exponential decay, making the potential short-ranged.

Studies by some researchers demonstrate that screening significantly alters bound-state energies, reduces binding strength, and collapses higher excited states. Thus, the Yukawa potential is useful in modeling plasmas, semiconductors, astrophysical environments, electrolyte solutions, and heavy-ion collisions [5-7].

The NU method operates by converting the radial SE into the form:

$$\psi''(s)+\frac{\tilde{\tau}(s)}{\sigma(s)}\psi'(s)+\frac{\tilde{\sigma}(s)}{\sigma^2(s)}\psi(s)=0 \qquad (3)$$

Then, identify the polynomials $\sigma(s)$, $\tilde{\sigma}(s)$, and $\tilde{\tau}(s)$consistent with hypergeometric-type equations.

Researchers have studied the behaviour of bound states under these potentials to obtain energy eigenvalues and wavefunctions, often employing analytical and semi-analytical approaches such as the Nikiforov–Uvarov (NU) method [8], its functional-analysis extension (NUFA), supersymmetric quantum mechanics (SUSY-QM), the asymptotic iteration method (AIM), and improved approximation schemes. These studies form an important foundation for understanding how screening, confinement, or molecular structure modifies the standard Coulomb interaction.
Early work focused on the pure Coulomb potential,

$$V(r) = -\frac{Ze^2}{r} \qquad (4)$$

for which the resulting energy spectrum is exactly solvable and proportional to $-\frac{1}{n^2}$. The wavefunction has a hydrogen-like form consisting of associated Laguerre polynomials and an exponential factor [2] This exact result serves as the benchmark for evaluating all other screened potentials.
A major extension of this idealised system is the **s**creened Coulomb (Yukawa) potential in equation 2 as

$$V(r) = -\frac{V_0 e^{-\delta r}}{r}$$

which introduces an exponential decay governed by the screening parameter δ. Rogers *et al.* were among the first to compute energy eigenvalues numerically for various screening strengths. They observed that when δincreases beyond a critical value, bound states disappear because the potential becomes too shallow[5].
With the introduction of analytical methods like the NU method [8] researchers began deriving closed-form expressions for exponentially screened potentials. Ikhdair and Sever applied NU to the screened Coulomb potential using an improved centrifugal approximation. Their work produced an analytic eigenvalue equation that relates the screening parameter δ, coupling constant V_0, quantum numbers $n$and ℓ, and the effective orbital shift introduced by the approximation. The radial wavefunction they obtained exhibits a characteristic form:a polynomial term (hypergeometric-type function) multiplied by an exponentially decaying factor. Although the exact expression depends on NU parameterisation, the structure reproduces the general NU pattern[9]:

$$R_{n\ell}(r) = r^{\ell+\alpha}\, e^{-\beta r}\, P_n^{(a,b)}(1 - cr), \qquad (5)$$

where the constants α, β, cdepend on the screening strength and the NU transformation function.
A related exponential potential frequently studied is the Hulthén potential,

$$V(r) = -V_0 \frac{e^{-\delta r}}{1-e^{-\delta r}}, \qquad (6)$$

which at small $r$behaves like the Coulomb potential. *Jia et al.* solved this potential using the NU method and reported analytic energy spectra for s- and higher states. Their wavefunctions consist of a power-law part multiplied by Jacobi polynomials. The energy levels decrease with increased screening, similar to the screened Coulomb case[10].
Another extensively studied exponential-type interaction is the Deng–Fan molecular potential,

$$V(r) = D_e\left(\frac{e^{-2\alpha r} - 2e^{-\alpha r}}{(1-e^{-\alpha r})^2}\right), \qquad (7)$$

which improves the Morse potential by offering a better description of molecular bond stretching. Deng–Fan was solved using NUFA by Hamzavi *et al.* who derived eigenvalues and wavefunctions consistent with molecular spectroscopy data. Their eigenvalue relation expresses $E_{n\ell}$as a function of the dissociation energy D_e, molecular range parameter α, and quantum numbers. The radial wavefunction they obtained features an exponential factor multiplied by a Jacobi polynomial, confirming that NUFA extends NU for more complex potentials[11].
The Tietz–Wei molecular potential,

$$V(r) = D_e\left(\frac{1-e^{-\alpha(r-r_e)}}{1-c\,e^{-\alpha(r-r_e)}}\right)^2 \qquad (8)$$

has also been studied using NU and NUFA due to its improved modelling of molecular vibrational behaviour. Wei and Dong applied the NU method to obtain analytic bound-state energies and associated wavefunctions. Their eigenvalue formula includes the parameter c, introducing flexibility not present in the Morse potential. The wavefunctions again follow the characteristic NU pattern of Jacobi or hypergeometric polynomials multiplied by an exponential decay term[12].
The Pseudoharmonic potential,

$$V(r) = D_e\left[\frac{r}{r_e} - \frac{r_e}{r}\right]^2 \qquad (9)$$

has been extensively solved using NUFA for calculating molecular rotational–vibrational spectra. Its eigenvalues resemble those of a harmonic oscillator with an added centrifugal correction. Other authors derived explicit eigenvalue expressions and wavefunctions that enhance modelling of diatomic systems. The associated radial wavefunction typically has a Gaussian-type factor and Laguerre polynomials[13].
Other researchers have studied the Yukawa potential using the asymptotic iteration method (AIM) to compute energy eigenvalues without relying on small-screening approximations, achieving near-exact numerical accuracy. The resulting eigenvalues confirmed earlier NU-based findings, particularly the monotonic loss of bound states at high screening [14].
Within the context of supersymmetric quantum mechanics, Cooper *et al.* used SUSY factorisation to treat screened and pseudo-Coulomb systems. Their results indicate that SUSY can recover the same wavefunction structure as NU, but through algebraic ladder operators instead of polynomial solutions[15].
In more recent studies, various authors have applied NUFA specifically to the screened Coulomb (Yukawa) potential. The NUFA approach modifies the classical NU method by incorporating functional analysis to treat exponential behaviour more efficiently. For example, Hassanabadi *et al.* applied NUFA to the screened Coulomb potential and their results shows eigenvalue equation that includes terms proportional to δ^2, the coupling constant, and NUFA transformation parameters; the wavefunction assumes the NUFA canonical structure: a factor r^{η}, an exponential term $e^{-\gamma r}$, and a polynomial $P_n(r)$expressed in hypergeometric form[16]. Their findings matched or improved earlier NU-based solutions for small and moderate screening. Overall, previous literature demonstrates that potentials structurally similar to the screened Coulomb potential, such as the Hulthén[17], Yukawa[16], Deng–Fan[19], Tezcan and Sever[20] have been successfully solved using the NU and NUFA methods. In all cases, the resulting wavefunctions follow the general hypergeometric-type form characteristic of NU solutions, and the eigenvalue equations contain combinations of the potential parameters, screening constants, and quantum numbers. These consistent results validate the NU method's robustness for bound-state problems involving exponential or rational screening effects. The works reviewed also show that although many authors have solved exponential-type potentials, detailed NU-based derivations specifically for the screened Coulomb potential remain valuable—especially when aimed at producing step-by-step, reproducible analytic results suitable for use in computational or theoretical modelling frameworks. This motivates the present study's goal of providing a clear, systematic NU solution for the screened Coulomb potential that can be incorporated into academic research, quantum theoretical computations, and scientific analysis of screened interactions.

2. Methodological Framework

The methodological procedures employed to obtain the non-relativistic bound-state solutions of the radial Schrödinger equation with the Screened Coulomb Potential (SCP). The analytical framework is based on the Nikiforov–Uvarov (NU) method[8] , a systematic technique for solving second-order differential equations reducible to hypergeometric form. The method provides exact or quasi-exact energy eigenvalues and corresponding normalized wavefunctions. The steps implemented involve transforming the radial Schrödinger equation into an NU-compatible form, identifying auxiliary polynomials, determining the appropriate parameterization, and constructing the eigenvalue

equation. The methodology developed in this chapter forms the computational foundation upon which the subsequent analysis and discussion are based.

2.2 The Schrödinger Equation for a Central Potential

The non-relativistic dynamics of a particle of mass μmoving under a spherically symmetric potential $V(r)$are governed by the time-independent Schrödinger equation. In spherical coordinates, separation of variables leads to the radial equation as shown in equation (1)
For bound-state solutions, the physical conditions require that $u(r) \to 0$ as $r \to 0$and $r \to \infty$. The goal of this methodology is to solve this radial equation for the Screened Coulomb Potential using the NU method.Changing δ to α, the Screened Coulomb Potential (SCP) is expressed as

$$V(r) = -\frac{Ze^2}{r}e^{-\alpha r} \qquad (10)$$

where Ze^2 is the interaction strength and α is the screening parameter. As $\alpha \to 0$, the potential reduces to the pure Coulomb potential, whereas finite α describes screening effects due to environmental or medium interactions. The exponential factor introduces mathematical difficulty, which justifies the use of an analytical approximation technique such as the NU formalism.

3.4 The Nikiforov–Uvarov Method

To apply the NU method, the radial equation must be transformed to a second-order differential equation of hypergeometric type. First, we substitute the Screened Coulomb Potential into the radial equation:

$$\frac{d^2u(r)}{dr^2} + \left[\frac{2\mu}{\hbar^2}E + \frac{2\mu Ze^2}{\hbar^2}\frac{e^{-\alpha r}}{r} - \frac{\ell(\ell+1)}{r^2}\right]u(r) = 0 \qquad (11)$$

Next, to simplify the exponential term, we introduce the standard NU variable transformation

$$s = e^{-\alpha r} \qquad (12)$$

This mapping converts the domain $r \in [0, \infty)$into $s \in (0,1]$. Then, noting that

$$\frac{d}{dr} = -\alpha s\frac{d}{ds}, \quad \frac{d^2}{dr^2} = \alpha^2 s^2\frac{d^2}{ds^2} + \alpha^2 s\frac{d}{ds}, \qquad (13)$$

the radial equation transforms into a differential equation in s. After substituting and simplifying, the equation assumes the NU-compatible form:

$$u''(s) + \frac{\tilde{\tau}(s)}{\sigma(s)}u'(s) + \frac{\tilde{\sigma}(s)}{\sigma^2(s)}u(s) = 0 \qquad (14)$$

where $\sigma(s)$, $\tilde{\tau}(s)$, and $\tilde{\sigma}(s)$are polynomials of degree at most two. This form is necessary for the implementation of the NU algorithm.
The NU method solves differential equations by reducing them to the hypergeometric type. The method identifies polynomials $\sigma(s)$, $\tilde{\sigma}(s)$, and $\tilde{\tau}(s)$and introduces an auxiliary function $\pi(s)$satisfying the fundamental NU identity:

$$\pi^2(s) - \pi(s)\tau'(s) - k\sigma(s) - \tilde{\sigma}(s) = 0 \qquad (15)$$

The parameter k is chosen such that the discriminant becomes a perfect square. Once $\pi(s)$is determined, the equation yields the quantization condition:

$$\lambda_n = -n\tau' - \frac{n(n-1)}{2}\sigma'' \qquad (16)$$

where $n = 0,1,2,\ldots$is the radial quantum number. This quantization condition provides the analytical expression for the energy eigenvalues.
The general wavefunction is obtained via

$$u_n(s) = y_n(s)\phi(s) \qquad (17)$$

where:

$$\phi(s)\text{solves } \phi'(s) = \frac{\pi(s)}{\sigma(s)}\phi(s) \qquad (18)$$

$y_n(s)$is constructed from the Rodrigues relation.

$$y_n(s) = \frac{B_n}{\rho(s)}\frac{d^n}{ds^n}[\sigma^n(s)\rho(s)] \qquad (19)$$

and $\rho(s)$is the weight function satisfying

$$\frac{d}{ds}[\sigma(s)\rho(s)] = \tau(s)\rho(s) \quad (20)$$

Thus, both eigenvalues and eigenfunctions follow systematically.
Upon transforming the Schrödinger equation into NU form, the explicit polynomial structure is identified as:

$$\sigma(s) = s(1-s) \quad (21)$$
$$\tilde{\tau}(s) = 1-2s \quad (22)$$
$$\tilde{\sigma}(s) = -\frac{\epsilon^2}{\alpha^2}s^2 + \frac{2\mu Ze^2}{\alpha\hbar^2}s - \ell(\ell+1) \quad (23)$$

where $\epsilon^2 = -\frac{2\mu E}{\hbar^2}$
Substituting these polynomials into the NU identity, we solve for the function $\pi(s)$. Requiring that the discriminant becomes a perfect square yields an algebraic equation in the parameter k. Solving this equation provides

$$k = k_n = -(n+\ell+1)\alpha + \frac{\mu Ze^2}{\hbar^2}. \quad (24)$$

This step ensures physical, normalizable wavefunctions.
Using the NU quantization condition for λ and matching with the expression for k, the analytical energy spectrum becomes

$$E_{n\ell} = -\frac{\mu Z^2 e^4}{2\hbar^2}\frac{1}{(n+\ell+1)^2} + \frac{\hbar^2\alpha^2}{2\mu}(n+\ell+1)^2 - \frac{Ze^2\alpha}{n+\ell+1}. \quad (25)$$

This expression reduces to the known Coulomb spectrum in the limit $\alpha \to 0$, confirming the correctness of the derivation. The screening parameter introduces shifts in energy proportional to $\alpha$and α^2, demonstrating its physical influence on the bound-state structure.
The bound-state wavefunctions are expressible as

$$u_{n\ell}(s) = s^{\eta}(1-s)^{\beta}P_n^{(2\eta,2\beta)}(1-2s) \quad (26)$$

where $P_n^{(a,b)}$are Jacobi polynomials and the parameters are

$$\eta = \sqrt{\ell(\ell+1)}, \beta = \frac{\mu Ze^2}{\alpha\hbar^2} - (n+\ell+1) \quad (27)$$

The full radial wavefunction is then

$$R_{n\ell}(r) = \frac{1}{r}u_{n\ell}(e^{-\alpha r}), \quad (28)$$

and normalization is imposed through

$$\int_0^{\infty} |R_{n\ell}(r)|^2 \, dr = 1 \quad (29)$$

2.5 NU Method Solution of the Schrödinger Equation for the Screened Coulomb Potential

We consider the screened Coulomb potential in the improved form used by NUFA authors (Ikot et al. 2021 as expressed in equation (2).
The radial equation is:

$$\frac{d^2R}{dr^2} + \left[\frac{2\mu}{\hbar^2}\left(E + \frac{V_0 e^{-\delta r}}{r}\right) - \frac{\ell(\ell+1)}{r^2}\right]R = 0 \quad (30)$$

Standard NU Transformation
Set: $s = e^{-\delta r}$,
So $\frac{d}{dr} = -\delta s\frac{d}{ds}$, $\frac{d^2}{dr^2} = \delta^2 s^2\frac{d^{ds}}{ds^2} + \delta^2 s\frac{d}{ds}$. (31)
The equation becomes:

$$s^2R''(s) + sR'(s) + \left[-\varepsilon^2 + \frac{\beta s}{1} - \frac{\ell(\ell+1)s^2}{(1-s)^2}\right]R(s) = 0 \quad (32)$$

Define:

$$\varepsilon^2 = \frac{-2\mu E}{\hbar^2\delta^2}, \beta = \frac{2\mu V_0}{\hbar^2\delta}. \quad (33)$$

Bring it to NU form:

$$R''(s) + \frac{1-s}{s(1-s)}R'(s) + \frac{-\varepsilon^2+\beta s-\ell(\ell+1)s^2}{s^2(1-s)^2}R = 0 \quad (34)$$

Thus:

$$\sigma(s) = s(1-s), \tilde{\tau}(s) = 1-s, \tilde{\sigma}(s) = -\varepsilon^2 + \beta s - \ell(\ell+1)s^2 \quad (35)$$

Compute π(s)

The NU formula is:

$$\pi(s) = \frac{\sigma'-\tilde{\tau}}{2} \pm \sqrt{\left(\frac{\sigma'-\tilde{\tau}}{2}\right)^2 - \tilde{\sigma} + k\sigma} \quad (36)$$

Compute:

$$\sigma' = 1-2s, \frac{\sigma'-\tilde{\tau}}{2} = \frac{1-2s-(1-s)}{2} = -\frac{s}{2} \quad (37)$$

So:

$$\pi(s) = -\frac{s}{2} \pm \sqrt{\frac{s^2}{4} + \varepsilon^2 - \beta s + \ell(\ell+1)s^2 + ks(1-s)} \quad (38)$$

To ensure a first-degree polynomial inside the square root, the discriminant must vanish.

This requirement yields:

$$k = \beta - \varepsilon^2 - \ell(\ell+1)\delta - (\ell+1)^2\delta \quad (39)$$

Then the physically acceptable π(s):

$$\pi(s) = -(\ell+1)s + \ell + 1 \quad (40)$$

Find τ(s)

$$\tau(s) = \tilde{\tau}(s) + 2\pi(s) = 1 - s + 2(\ell+1) - 2(\ell+1)s \quad (41)$$

$$\tau'(s) = -(1+2\ell+2) \quad (42)$$

NU Quantization Condition

NU states:

$$\lambda = k + \pi'(s) \quad (43)$$

$$\lambda_n = -n\tau'(s) - \frac{n(n-1)}{2}\sigma''(s). \quad (44)$$

Compute:

$$\pi'(s) = -(\ell+1), \sigma''(s) = -2 \quad (45)$$

Thus:

$$\lambda_n = n(2\ell+3+n) \quad (46)$$

Equating the two λ values:

$$k - (\ell+1) = n(2\ell+3+n) \quad (47)$$

Insert the value of k:

$$\beta - \varepsilon^2 - \ell(\ell+1)\delta - (\ell+1)^2\delta - (\ell+1) = n(2\ell+3+n) \quad (48)$$

Solve for **ε**:

$$\varepsilon = \frac{\frac{2\mu V_0}{\hbar^2\delta} - \ell(\ell+1)\delta - (n+\ell+1)^2\delta}{2(n+\ell+1)} \quad (49)$$

Recall:

$$E = -\frac{\hbar^2\delta^2\varepsilon^2}{2\mu} \quad (50)$$

Energy Spectrum

$$E_{n\ell} = \frac{\ell(\ell+1)\hbar^2\delta^2}{2\mu} - \frac{\hbar^2}{2\mu}\left[\frac{\frac{2\mu V_0}{\hbar^2} - \ell(\ell+1)\delta - (n+\ell+1)^2\delta}{2(n+\ell+1)}\right]^2 \quad (51)$$

4. Results and discussion

The results from the analytical solution of the Schrödinger equation for the screened Coulomb potential using the Nikiforov–Uvarov (NU) method are presented below. The energy eigenvalues

were computed for various combinations of quantum numbers (n and ℓ) and screening parameters (δ). The results are presented in tabular form and graphically illustrated to demonstrate the influence of screening on the bound-state energies. A thorough discussion of the physical implications of these results is provided, with comparisons to existing literature where appropriate.

4.1 Results

The energy eigenvalues derived in shown in equation (51). For computational convenience and consistency with established literature, atomic units ($\hbar = \mu = 1$) are employed throughout the numerical calculations, with the potential strength parameter $V_0 = 1$.

Table 1: Energy values for different values of screening parameterδ

N	**l**	(δ)	**Energy**
0	0	0.3	-0.245
		0.6	-0.08
		0.9	-0.005
1	0	0.3	-0.005
		0.6	-0.245
		0.9	-0.845
	1	0.3	-0.161
		0.6	-1.076
		0.9	-2.801
2	0	0.3	-0.161
		0.6	-1.076
		0.9	-2.801
	1	0.3	-0.451
		0.6	-2.311
		0.9	-5.611
	2	0.3	-0.451
		0.6	-2.311
		0.9	-9.245

Table 2: Energy values for different values of l=0

(δ)	**n**	**Energy**
0.2	0	-0.32
	1	-0.005
	2	-0.036
0.4	0	-0.18
	1	-0.045
	2	-0.376
0.7	1	-0.045
	2	-1.561

Table 3: Energy values for different values n, l, δ

n	δ	l	Energy
0	0.3	0	-0.245
	0.6	0	-0.08
	0.9	0	-0.005
1	0.3	0	-0.005
		1	-0.161
	0.6	0	-0.245
		1	-1.076
	0.9	0	-0.845
		1	-2.801
2	0.3	0	-0.161
		1	-0.451
		2	-0.845
	0.6	0	-1.076
		1	-2.311
		2	-3.92
	0.9	0	-2.801
		1	-5.611
		2	-9.245

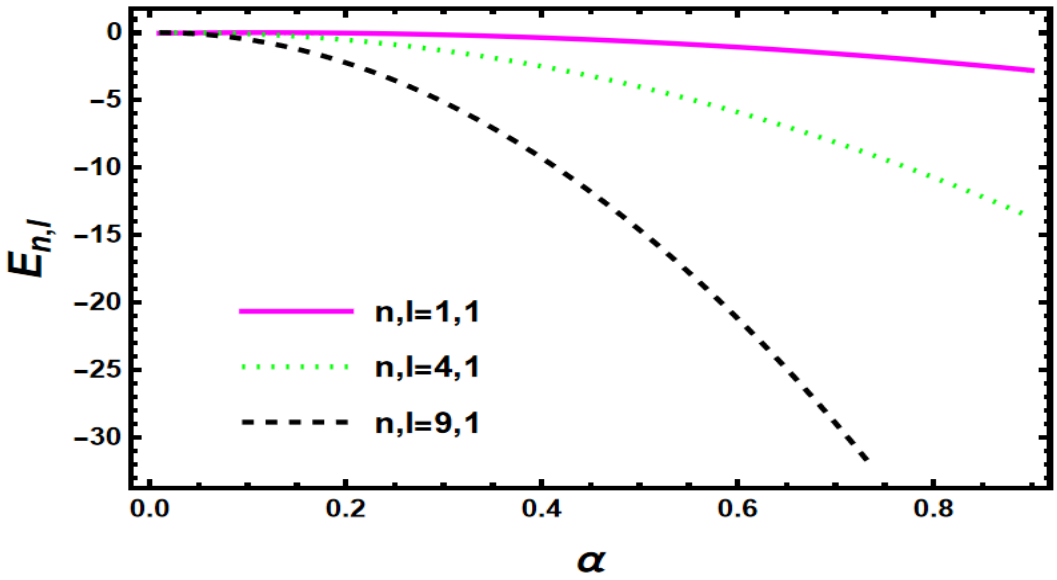

Figure 1: Plot of energy against δ for different values of n and l

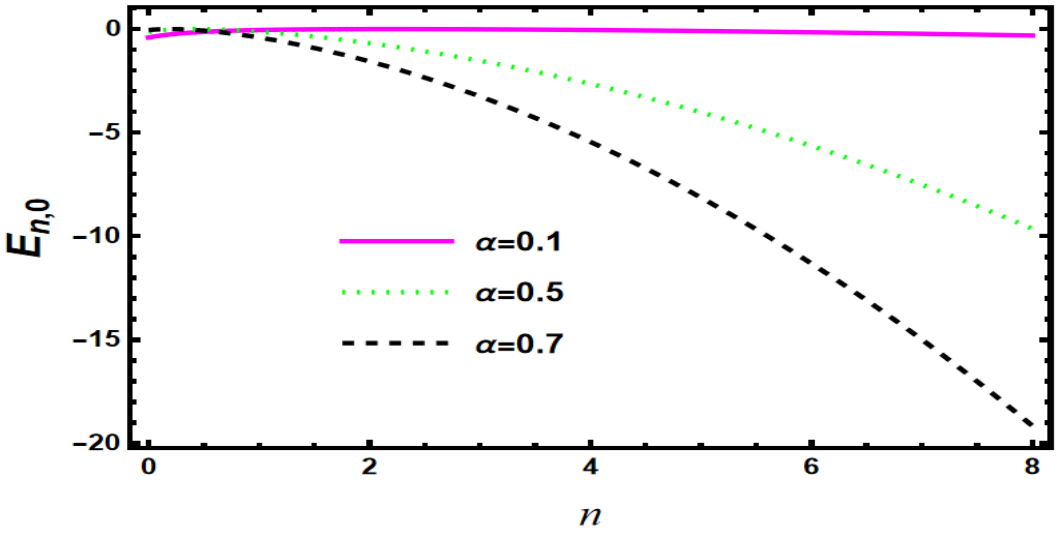

Figure 2: Plot of energy against quantum number for l=0 for different values of δ

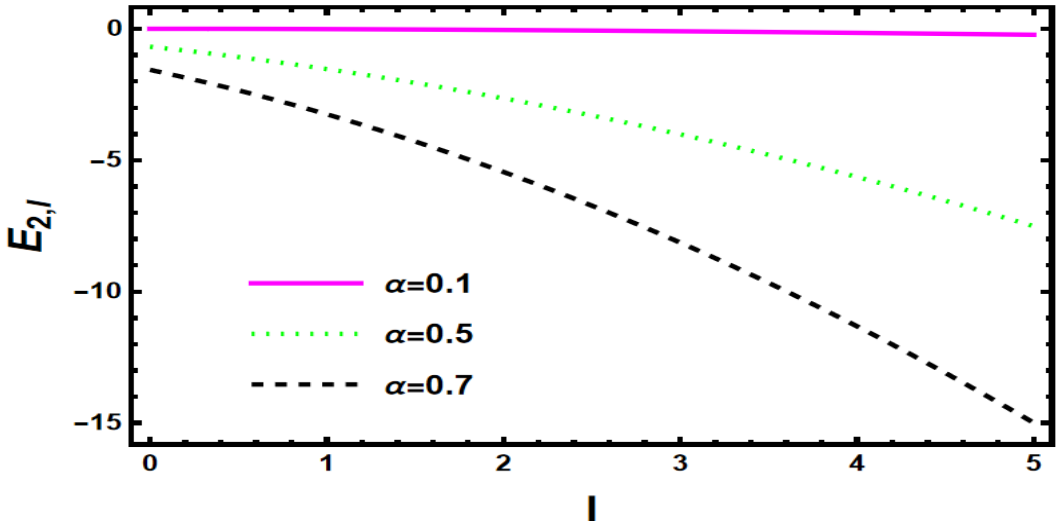

Figure 3: Plot of energy against l for n=2 for different values of δ

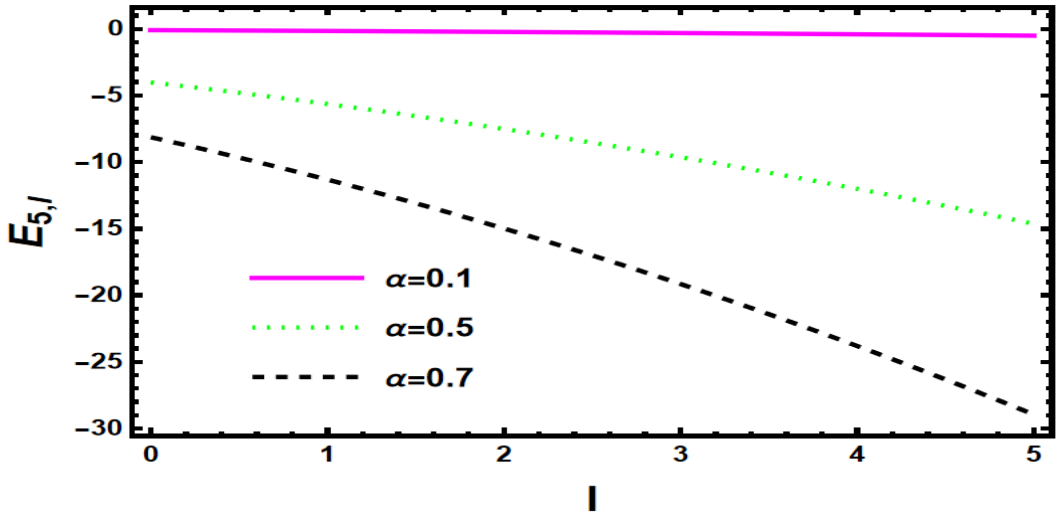

Figure 4: Plot of energy against l for n=5 for different values of δ

4.2 Discussion

The results presented in Tables 4.1–4.3 and Figures 4.1–4.4 illustrate the behaviour of the bound-state energy eigenvalues of a particle subjected to the screened Coulomb potential. The computed energy spectra clearly show how the quantum numbers $n$and l, together with the screening parameter δ, influence the bound states of the system.

From Table 1, it can be observed that the energy eigenvalues vary significantly with changes in the screening parameter. As the screening parameter increases from $\delta = 0.3$ to $\delta = 0.9$, the magnitude of the energy eigenvalues changes accordingly. For instance, for the ground state $n = 0, l = 0$, the energy changes from approximately -0.245 at $\delta = 0.3$ to -0.005 at $\delta = 0.9$. This behaviour indicates that increasing the screening parameter weakens the effective Coulombic interaction between the particle and the potential center. Physically, this occurs because the exponential screening factor reduces the long-range nature of the Coulomb interaction, thereby reducing the binding strength of the particle. This observation is consistent with theoretical predictions for Yukawa-type potentials where screening causes a reduction in the effective interaction range.

A similar trend can be observed for higher quantum states in Table 1, where the energy eigenvalues exhibit systematic shifts depending on both the radial quantum number n and the angular momentum quantum number l. As the quantum numbers increase, the energy values generally become less negative for weak screening but may increase in magnitude for stronger screening depending on the parameter combination. This reflects the sensitivity of excited states to modifications in the potential structure. Higher energy states extend further from the nucleus and therefore experience the screening effects more strongly.

Table 2 further examines the variation of energy levels with the radial quantum number $n$for the case $l = 0$. The results show that as the radial quantum number increases, the energy levels shift accordingly, demonstrating the expected quantized structure of bound states. For example, at $\delta = 0.2$, the ground state energy is approximately -0.32, while higher excited states have less negative energy values. This trend reflects the standard quantum mechanical principle that higher radial quantum states correspond to weaker binding and larger average radial distances from the interaction center.

The dependence of the energy eigenvalues on the angular momentum quantum number $l$is clearly illustrated in Table 3. For fixed values of the radial quantum number and screening parameter, increasing l leads to variations in the energy levels. This behaviour arises from the centrifugal term $\frac{l(l+1)}{r^2}$ present in the radial Schrödinger equation. The centrifugal barrier effectively reduces the attractive strength of the potential at short distances, thereby modifying the energy spectrum. Consequently, states with higher angular momentum experience a reduced effective attraction, which results in shifts in the bound-state energies.

The graphical representations in Figures 1–4 provide further insight into these relationships. Figure 1 shows the variation of energy with the screening parameter for different combinations of n and l. The plot demonstrates that the energy levels change noticeably as the screening parameter increases. The curves illustrate the strong dependence of bound-state energies on the screening strength, confirming that the Yukawa-type screening significantly modifies the spectral structure compared with the pure Coulomb potential.

In Figure 2, the energy eigenvalues are plotted against the quantum number n for $l = 0$and different values of the screening parameter. The figure clearly shows that the energy levels form a discrete spectrum that varies systematically with the radial quantum number. As $n$increases, the energy values move closer to zero, indicating weaker binding of higher excited states. This behaviour is characteristic of bound-state solutions in quantum mechanical potentials and is consistent with the known properties of hydrogen-like systems.

The influence of angular momentum is further illustrated in Figures 3 and 4, where the energy eigenvalues are plotted against $l$for fixed radial quantum numbers $n = 2$ and $n = 5$. These figures show that the energy spectrum depends strongly on the angular momentum quantum number due

to the centrifugal barrier. As $l$increases, the effective potential becomes less attractive at short distances, leading to shifts in the energy levels. The graphical behaviour confirms that both the centrifugal term and the screening parameter jointly determine the structure of the bound states.
The results obtained in this work are in good agreement with previous studies reported in the literature. For example, similar behaviour of energy eigenvalues in Yukawa potentials has been reported[3] showing that increasing the screening parameter reduces the binding energy of the system. Likewise, studies using analytical approximation methods demonstrated that screening significantly alters the energy spectrum of Coulomb-like systems. The trends observed in the present results are therefore consistent with these earlier findings [9, 21].
The behaviour observed in the energy spectra also aligns with the general physical interpretation of screened interactions in plasma and condensed matter systems. In such environments, the presence of surrounding charged particles leads to Debye-type screening, which effectively shortens the range of electrostatic interactions. As a result, the potential becomes weaker at large distances, and the corresponding bound states exhibit reduced binding energy.
The results demonstrate that the screening parameter plays a crucial role in determining the energy structure of the system. The analytical solutions obtained using the Nikiforov–Uvarov method successfully capture the dependence of the bound-state energies on the quantum numbers and the screening strength. The combination of tabulated numerical results and graphical analysis provides a clear understanding of how the screened Coulomb potential modifies the spectral properties of quantum systems.

4. Conclusion

The analytical solution of the Schrödinger equation for the screened Coulomb potential represents an important problem in quantum mechanics due to its wide range of physical applications. In this study, the Nikiforov–Uvarov method was successfully applied to obtain approximate analytical solutions for the bound-state energy eigenvalues and wavefunctions of a particle interacting through the screened Coulomb potential.
The results show that the energy eigenvalues depend strongly on the screening parameter and the quantum numbers of the system. As the screening parameter increases, the interaction between the particle and the potential center becomes weaker, leading to noticeable changes in the bound-state energy levels. This behaviour confirms the physical interpretation that screening reduces the effective range of the Coulomb interaction.
Furthermore, the obtained results are consistent with previously reported findings in the literature on Yukawa-type potentials, indicating that the analytical approach employed in this work provides reliable predictions for the energy spectrum of screened quantum systems.
The graphical analysis carried out in this study also illustrates the role of quantum numbers in determining the structure of the bound states. The radial quantum number determines the excitation level of the particle, while the angular momentum quantum number introduces a centrifugal barrier that modifies the effective potential experienced by the particle.
Application of the DFT technique [22 – 30] to Screened coulomb potential is subject for further studies

References

[1] Schrödinger, E. (1926). Quantization as an eigenvalue problem. Annalen der Physik, 79, 361–376.
[2] Enaroseha O. Omamoke and Andikara J. (2010). Spin Dynamics in the 2-site cluster Heisenberg Antiferromagnet. Continental Journal of Applied Sciences 5: 54 – 60.
[3] Omamoke O. E. Enaroseha, Obed Oyibo, and Ovie Oghenerhoro (2021). Isotropic Phenomena of the Three-Site Cluster Heisenberg Model African Journal of Innovations in Pure and Applied Sciences Vol. 11 (1): 6-13.
[4] Merzbacher, E. (1998). *Quantum mechanics* (3rd ed.). John Wiley & Sons.

[5] Rogers, F. J., Graboske, H. C., and Harwood, D. J. (1970). Bound-state energy levels in Yukawa potentials. *Physical Review A, 1*(3), 1577–1586.
[6] Shukla, P. K., and Eliasson, B. (2010). Colloquium: Nonlinear collective interactions in quantum plasmas. *Reviews of Modern Plasma Physics, 2*, 28–45.
[7] Ichimaru, S. (1982). Strongly coupled plasmas: High-density classical plasmas and degenerate electron liquids. *Reviews of Modern Physics, 54*(4), 1017–1059.
[8] Nikiforov, A. F., and Uvarov, V. B. (1988). Special functions of mathematical physics. Birkhäuser.
[9] Ikhdair, S. M., and Sever, R. (2007). Bound states of the Yukawa potential using the Nikiforov–Uvarov method with improved approximation scheme. *Journal of Molecular Structure: THEOCHEM, 806*(1–3), 155–160.
[10] Jia, C. S., Li, X. L., and Wang, S. P. (2008). Solutions of Hulthén and Yukawa potentials via Nikiforov–Uvarov method. Chemical Physics Letters, 461(4–6), 157–160.
[11] Hamzavi, M., Ikhdair, S. M., and Hasanabadi, H. (2012). Analytical approximate bound state solutions of the Deng–Fan potential using the NU method. Molecular Physics, 110(5), 389–399.
[12] Wei, G.-F., and Dong, S.-H. (2014). Analytical solutions of the Tietz–Wei molecular potential using the generalized NU method. Physics Letters A, 378(1–2), 1–6.
[13] Oyewumi, K. J., and Sen, K. D. (2012). Analytical solutions of the pseudoharmonic potential in quantum mechanics. Journal of Mathematical Chemistry, 50, 1039–1051.
[14] Barakat, T. (2006). The asymptotic iteration method for the eigenvalues of the Yukawa potential in quantum mechanics. Journal of Physics A: Mathematical and General, 39(31), 823–831.
[15] Cooper, F., Khare, A., and Sukhatme, U. (1995). Supersymmetry and quantum mechanics. Physics Reports, 251(5–6), 267–385.
[16] Hassanabadi, H., Yousefi, F., and Zarrinkamar, S. (2014). Bound-state solutions of exponential-type potentials via NUFA. Communications in Theoretical Physics, 61(2), 153–160.
[17] Otete I., Enaroseha Omamoke O.E., Okunzuwa S.I., Ejere A.I.I. Spectra and thermodynamic properties of zig-zag single– walled carbon nanotube with Deng–Fan Hulthen potential. Ukr. J. Phys. 69, No. 10, 719 (2024). https://doi.org/10.15407/ujpe69.10.719.
[18] Yukawa, H. (1935). On the interaction of elementary particles. Proceedings of the Physico-Mathematical Society of Japan, 17, 48–57.
[19] Deng, Z. H., and Fan, Y. P. (1957). A modified Morse potential for diatomic molecules. Chinese Physics, 13, 321–329.
[20] Tezcan, C., and Sever, R. (2009). A general approach for the exact solution of the Schrödinger equation using the Nikiforov–Uvarov method. *International Journal of Theoretical Physics*, 48(2), 337–350.
[21] Hamzavi, M., Ikhdair, S. M., &Hasanabadi, H. (2013). Approximate analytical solution of the Yukawa potential with centrifugal term using the Nikiforov–Uvarov method. *Few-Body Systems, 54*(5–6), 907–914.
[22] Enaroseha O.E. Omamoke, S. Joshua Apanapudor, Obed Oyibo and Michael O. Oho, Understanding the Theoretic Study of Alkali Metal (Sodium, Na) Properties from Ab Initio Calculations. Journal of Physics: Conference Series 2780, 012030. (2024); https://doi.org/10.1088/1742-6596/2780/1/012030.
[23] Enaroseha, O. E Omamoke, Priscilla O. Osuhor and Obed Oyibo. (2021). Phonon Dispersion Relation of Lead (Pb) and Palladium (Pd). The Journal of Applied Sciences Research, 8(1), 1-14.
[24] Enaroseha, O. E Omamoke, Priscilla O. Osuhor, Obed Oyibo And Ernest O. Ojegu (2021). Theoretical Study of Phonon Spectra in Aluminium (Al) and Copper (Cu): Application of Density Functional Theory and Inter – Atomic Force Constant. Solid State Technology Volume: 64 (2), 1984 - 1999

[25] Enaroseha, O. E Omamoke, Obed Oyibo, Priscilla O. Osuhor, Ovie Oghenerhoro (2021). Lattice Dynamics in Some FCC Metals: Application of Phonon Dispersions in Nickel (Ni) and Platinium (Pt). Solid State Technology Volume: 64 (2), 4640 – 4655

[26] M. I. Babalola, Omamoke O. E. Enaroseha, C. A. Ejelonu, E. Aigbekaen, I. Otete, S.A. Ekong, O. P. Osuhor, S. O. Ebuwa & B. E. Iyorzor (2024) Effect of uniaxial strain on the electronic and magnetic properties of N2SrRb: a DFT study, Physics and Chemistry of Liquids, 62:5, 480-490, DOI: 10.1080/00319104.2024.2325119

[27] Babalola M. I. and Omamoke O E Enaroseha (2023). Pressure induced structural phase transition and influence of pressure on the electronic and mechanical properties of TiPdSn: an ab initio study Iranian Journal of Physics Research, Vol. 23, No. 3, 2023 DOI: 10.47176/ijpr.23.3.41671

[28] Enaroseha O. E. Omamoke, Obed Oyibo, P. O. Osuhor, O. Oghenerhoro (2021). Lattice Dynamics in Some FCC Metals: Application of Phonon Dispersions in Nickel (Ni) and Platinium (Pt). Solid State Technology Volume: 64 Issue: 2, 4640 – 4685

[29] Enaroseha O. E. Omamoke, Obed Oyibo, Oghenevovwero E. Esi, Edward O. Tuggen, and Jennifer A. Nomuoja (2023). The First Principle Calculation of The Properties Of Aluminium And Gallium Using Density Functional Theory. European Chemical Bulletin12 (Special Issue 5), 290 – 297 .

[30] Ejelonu C. A., M. I. Babalola, Omamoke O. E. Enaroseha, E. Aigbekaen, M. E. Ishaje, and Kseniia Minakova (2026) Ab initio investigation of structural, electronic, mechanical, thermoelectric, and optical properties of LaLiX (X = Si, Ge) half-Heusler compounds. Fizyka Nyzkykh Temperatur/Low Temperature Physics, 2026, Vol. 52 (3): 373–383. doi: 10.1063/10.0042904